Preliminary and Tentative Course Outline for: Unit Root and Cointegration Analysis

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The outline is **tentative and may be changed according to time available, needs and interests** and most likely not all material will be covered, i.a. depending upon the knowledge and interests of students and the rate of progress we achieve in the course. We only have 10 units and the material listed in the outline would allow to spend much more time – thus there will be compromises, imprecisions and shortcuts. I will, however, throughout try to give you detailed references where material that we do not cover in detail is discussed in detail.

1. Introduction
   - What is it about?
   - What is planned?
2. Stationary Processes
   - Stochastic processes and stationarity
   - White noise, autoregressive (AR), moving average (MA), autoregressive moving average processes, causality, invertibility
   - Multivariate ARMA processes
   - Estimation of mean and covariances
   - Parameter estimation for AR (and ARMA) processes
   - Further topics (model selection, state space representation, forecasting,...)
3. Univariate Integrated Processes and Unit Root Tests

- Integrated processes
- The functional central limit theorem ('working with Brownian motions')
- Testing for unit roots (and testing for stationarity)
- Further topics (structural breaks or unit roots, seasonal unit roots, higher integration orders)

4. Multivariate Integrated Processes and Cointegration

   - The triangular representation: the cointegrating regression
   - Estimation: OLS, Fully Modified OLS, Dynamic OLS, Integrated Modified OLS
   - Testing for cointegration: residual based tests
   - Further topics (monitoring,...)

5. Cointegration Analysis with Vector Autoregressive Models: The I(1) Case

   - Granger representation theorem
   - Johansen's ML - Reduced rank regression approach: parameter estimation and testing for the cointegrating rank
   - Further topics (the role of deterministic components, weak exogeneity,...)

6. Structural Vector Autoregressive Models

   - Stationary Case: Different types of structural models/relationships
   - Cointegrated Case: The structural vector error correction model
   - Estimation
   - Structural impulse response functions (IRFs) and forecast error variance decompositions (FEVDs)
   - Further topics (sign restrictions, factor augmented VAR models)

Remark 1 *An important aspect of econometric analysis is to actually do it. We will incorporate a hands-on component in the course, with many of the standard methods implemented in public domain software such as,* `EViews`, `gretl` *or* `JMULTI`.

More specific things that are not available in ready to use software typically need to be programmed by oneself. I will discuss some of these things using my own `MATLAB` *code. Part of the exercises could be – again depending upon interests – translation of parts of this code to e.g.* `R`.

Organization of the Course and Grading

- In addition to the lectures I will hand out some exercise sheets. In these exercises you are asked to solve some 'pencil and paper' exercises as well as to work on some computer related questions. The computer part itself will consist of two types of questions, one where I will ask you to program or simulate some small things – like finite sample distributions of tests or estimators or some critical values – and a second in which you will be asked to perform some econometric analysis (most likely using some ready made software) and some economic data, like the ones distributed.

- The format of the final exam will be discussed in the first unit of the course (oral or written classical exam, a short paper related to the material,...)

- The final grade depends upon the performance in both the exercises as well as in the exam.

Lecture Notes and Literature

The material is provided in the form of slides (where I will be grateful for the detection of typos etc.). In addition to giving you detailed references I will most likely distribute some papers. For your convenience I just list some books at the end of this document that are either classic or widely-used and briefly discuss here what is specific about these books in my view. The list of books and even more so the list of references (containing a wide list of papers on unit root and cointegration analysis) provided are not at all meant such that you have to read all this material.[1] I give you a wide list of papers so that you can, in case you wish to do so, read some underlying original material or have the precise references when I refer to some paper.[2]

- Banerjee et al. (1993): This books covers in an illustrative way many aspects of unit root and cointegration analysis. This is a good starting point if you want to get a 'feeling' for the issues arising in nonstationary time series econometrics. It is not a reference book if you want to go into great detail into a specific topic (but for that it provides many references).

- Brillinger (1981): Classical book with quite some focus on spectral analysis (including discussions of Fourier transforms) and estimation of spectra. The book also covers

[1] In particular I am not intending that you need to buy some or all of these books.

[2] For the empirical examples, the references will be provided when discussing these examples.

principal components and canonical analysis. The book, however, focuses on stationary time series.

- Brockwell and Davis (1991): A widely used classical textbook for stationary time series analysis, which is a necessary prerequisite to talk about integrated processes. Includes discussions of Hilbert spaces and of the relations between time and frequency domain. The book also contains detailed discussions concerning parameter estimation in uni- and multivariate AR and ARMA models.

- Davidson (1994): This is one of the most widely used asymptotics textbook in the unit root and cointegration community. It contains detailed discussions of various sets of assumptions (mixingales, near-epoch dependence,...) leading to Central Limit Theorems and Functional Central Limit Theorems. For those interested in details, it contains proofs of many results.

- Hannan and Deistler (1988): A classical high-level book about ARMA(X) and State Space Models for stationary time series. Very densely written and at a high level of abstraction.

- Johansen (1995): The book length treatment of cointegration analysis in vector autoregressive models by the main developer of this literature. It is a quite self-contained treatment of this topic. It does not contain a discussion of more 'practical' aspects like structural analysis (structural VARs and impulse response functions etc.). Contains proofs of the results in a unified notation.

- Juselius (2006): Something like an updated and more applications oriented companion of the previous book. More focus on the different 'things' one wants to do to 'identify' VAR models in some 'structural' sense.

- Lütkepohl (2005): The first part of the book covers many aspects of modeling stationary time series with VAR models and in the second part the extension to cointegration is treated. It also includes, compared to the book by Johansen, aspects relevant for economists, like structural VAR models etc. In the third part extensions are considered (VARMA instead of VAR, GARCH,...) The book also contains some appendices that contain useful material.

- Lütkepohl and Krätzig (2004): The 'applied' counterpart of the previous book which in addition covers smooth transition regression models. The editors are the developers of the free software `JMULTI` and all the examples from the book can be replicated with that software.

- Martin et al. (2013): This book covers a wide array of topics including – by construction – also more recent topics. An advantage of the book is that `R` and `MATLAB` code to replicate the examples is available online.

References

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