Functional regression analysis using **R**

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Examples

What are functional data?

- Activity and disease patterns
  (eg. monitoring birds, children or insects over time)
- Animal and human growth curves
  (eg. weight gain in pigs and dietary studies)
- Fluorescence curves
  (eg. photosynthesis processes over time (Ritz and Streibig, 2008))
- Reproduction histories
  (eg. longevity of medflies (Chiou et al, 2003))

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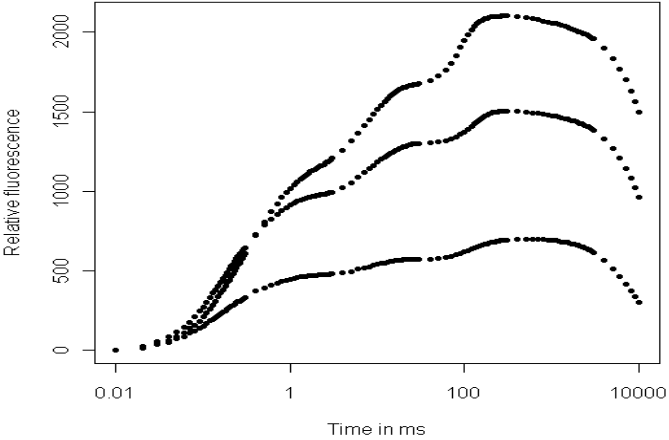
More about fluorescence curves

- Experiment:
  - dark-adapted leaves exposed to light
    (only the first seconds of this process is recorded!)
- Functional response:
  - proportion of light not used in the photosynthesis
- High throughput measurements:
  - fast and non-invasive
  - informative long before visual effects
- Curve trajectory changes with species and stress level

Observed fluorescence curves

Three *replicates*



More about functional data

Common features:

- ***repeated measurements*** on the same subject or unit
- basic observation: ***smooth function***
  (in practice observed discretely on a grid)

Use of functional data:

- classification/clustering
- ANOVA- and regression-like models
- prediction

Smoothness being exploited in various ways

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Functional regression

How to relate functional responses to scalar, explanatory variables?

Available functional regressions models:

- Semi-parametric approaches:
  - additive effects models (Ramsay & Silverman, 2005) (**R** package `fda` on CRAN and R-Forge)
  - multiplicative effects models (Chiou *et al.*, 2003) (**R** package `fmer` soon on CRAN)
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Functional multiplicative effects models

A little notation:

- $y_i : T \mapsto \mathbb{R}$ is a *function* ($i = 1, \ldots, N$)
- $T \subseteq \mathbb{R}$ is the interval
- Observed at points $t_1, \ldots, t_K$ (K large)

Multiplicative effects regression model:

$$E(y_i(t)|z_i) = \psi(t, z_i)\mu(t)$$

Right-hand side:

- μ: capturing the overall average trend
- ψ: multiplicative effects: low-degree polynomials in t with coefficients depending on explanatory variable z_i

Estimation – in two steps

1. Non-parametric estimation:
   - μ: *smoothing based on all curves* (**R** package `KernSmooth`)
   - coefficients in ψ: obtained using least squares

2. Parametric or semi-parametric estimation for coefficients:
   1. choose GLM (`glm()`) or quasi-likelihood model
   2. iterative estimation: (IWLS+smoothing)
      - ★ link and/or variance functions (not in GLM case)
      - ★ parameters in linear predictor

Using R

```
library(fmer)

bo.m1 <- fmerm(fluo2 ~ log(time), id2, id0,
data = barleyOat, quad = TRUE)
```

Arguments to `fmerm`:

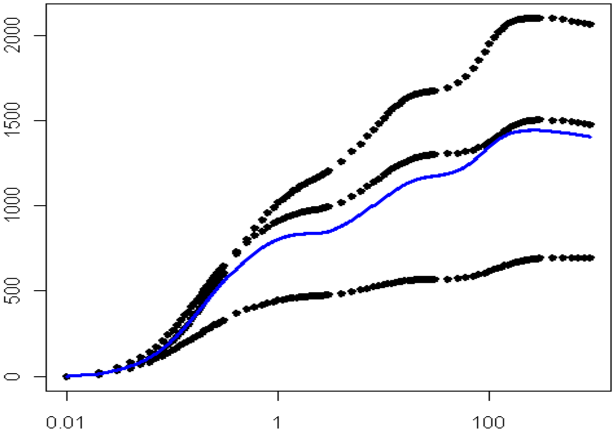
- `fluo2`: function values
- `log(time)`: grid values
- `id2`: curve id (54 curves in total)
- `id0`: treatment factor
- `quad`: ψ quadratic in t

Model fit components

- Estimated overall mean
- Estimated regression curves
  (use `plot` method)
- For each coefficient in ψ:
  - estimated link and variance functions
  - estimated parameters
    (use `summary` method)
  - fitted values and residuals
    (use `fitted` and `residuals`)

Fitted fluorescence curve

Using the `plot` method:



Pros and cons

- Advantages:
  - non-parametric modelling of *the form* of the curves (separating the time effect from other effects)
  - parametric regression models for *the differences* between curves
  - graphical model check available (`ratioPlot`)
- Drawbacks:
  - automatic bandwidth selection needed (used repeatedly)
  - two-step estimation procedure (some variation lost)

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Future R work

- Testing on more datasets!!!
- Setting up a modular structure for model fitting:
  - one function per step in estimation procedure
  - plug-ins for different smoothing methods
  - choice between bandwidth selection methods
  - more flexible model specification
- Constructing extractors for various fit components

Future theoretical work

- Joint estimation
- Extended modelling including the residual process
- Model checking diagnostics

References

Chiou, J. M., Müller, H.-G. and Wang, J. L. (2003).
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Ramsay, J. O. and Silverman, B. W. (2005).
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