M-estimators for structured covariance models

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In this talk, robust estimation of the covariance matrix is considered whenever
constrains are placed on the covariance matrix. Such models are particularly
important whenever there is low or insufficient sample support (small n, large p).

Graphical models overly the class of elliptical distributions are first considered. The
robust estimators considered here are the graphical M-estimators and the plug-in
M-estimators. The graphical M-estimators, which are newly introduced here, refer
to estimators obtained by optimized a robust loss criterion over the restricted
scatter structures imposed by a graphical model, whereas the plug-in M-
estimators refer to the estimators obtained by substituting an M-estimate of
scatter (or any other robust estimate of scatter) for the sample covariance matrix in
classical algorithm for the Gaussian graphical model. It turns out that, under
suitable conditions, both approaches yield the same asymptotic efficiency. For
relatively small sample sizes, however, the graphical M-estimator is more robust
and more efficient than the plug-in M-estimator. The research is joint with Daniel
Vogel.

Next, soft modeling or regularization is considered. Here, a general class of
regularized M-estimators for scatter is proposed. This class constitutes a natural
generalization of M-estimators of the scatter matrix and are defined as a solution
to a penalized M-estimation cost function. Using the concept of geodesic convexity,
the uniqueness of the regularized M-estimators of scatter and the uniqueness of
the solution to the corresponding M-estimating equations are established. An
iterative algorithm with proven convergence to the solution of the regularized M-
estimating equation is also given. Furthermore, we derive a simple, closed form
and data dependent solution for choosing the regularization parameters based on
shape matrix matching in the mean squared sense.

Finally, some simulations studies illustrate the improved accuracy of the proposed
regularized M-estimators of scatter compared to their non-regularized
counterparts in low sample support settings. This research is joint with Esa Ollila of
Aalto University, Finland.