The Long March Towards Joint Asymptotics: My 1st Steps...

We consider a joint asymptotic framework for studying the semi-nonparametric models, where (finite dimensional) Euclidean parameters and (infinite dimensional) functional parameters are both of interest. A class of partially linear models (under penalized quasi-likelihood estimation) is used as a prototypical example. Our first aim is to provide deep theoretical insights on the new joint asymptotics phenomenon. We first show that the Euclidean estimator and functional estimator after different re-scalings jointly converge to a Gaussian vector. This weak convergence result surprisingly discloses that these two estimators become asymptotically independent while the Euclidean estimator achieves the semiparametric efficiency bound. We next consider testing a set of joint local hypothesis based on the likelihood ratio approach. A semi-nonparametric version of Wilks phenomenon is unveiled. Our second aim is to develop more useful joint global inference for the same class of models. In particular, we find that "the inclusion of a faster convergent parametric estimator indeed affects the nonparametric inference although not at the convergence rate level." This conclusion is against the common intuition that the parametric term can be treated as if it were known given its faster convergence rate. Our joint asymptotics results are very challenging to establish due to the different natures of two model components: parametric v.s. nonparametric.