M-estimators for structured covariance models

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In this talk, robust estimation of the covariance matrix is considered whenever constrains are placed on the covariance matrix. Such models are particularly important whenever there is low or insufficient sample support (small n, large p).

Graphical models overly the class of elliptical distributions are first considered. The robust estimators considered here are the graphical M-estimators and the plug-in M-estimators. The graphical M-estimators, which are newly introduced here, refer to estimators obtained by optimized a robust loss criterion over the restricted scatter structures imposed by a graphical model, whereas the plug-in M-estimators refer to the estimators obtained by substituting an M-estimate of scatter (or any other robust estimate of scatter) for the sample covariance matrix in classical algorithm for the Gaussian graphical model. It turns out that, under suitable conditions, both approaches yield the same asymptotic efficiency. For relatively small sample sizes, however, the graphical M-estimator is more robust and more efficient that the plug-in M-estimator. The research is joint with Daniel Vogel.

Next, soft modeling or regularization is considered. Here, a general class of regularized M-estimators for scatter is proposed. This class constitutes a natural generalization of M-estimators of the scatter matrix and are defined as a solution to a penalized M-estimation cost function. Using the concept of geodesic convexity, the uniqueness of the regularized M-estimators of scatter and the uniqueness of the solution to the corresponding M-estimating equations are established. An iterative algorithm with proven convergence to the solution of the regularized M-estimating equation is also given. Furthermore, we derive a simple, closed form and data dependent solution for choosing the regularization parameters based on shape matrix matching in the mean squared sense.

Finally, some simulations studies illustrate the improved accuracy of the proposed regularized M-estimators of scatter compared to their non-regularized counterparts in low sample support settings. This research is joint with Esa Ollila of Aalto University, Finland.