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TITLE:

On Bayesian estimation of thermal diffusivity in materials

ABSTRACT:

Two approaches are presented to estimate the thermal conductivity or diffusivity of a homogeneous material from the temperature evolution acquired in few internal points. Temperature evolution is described by the classical one-dimensional heat equation, in which the thermal conductivity (or diffusivity) is one of the coefficients.

In the first approach noisy measurements lead to a partial differential equation with stochastic coefficients and, after discretisation in time and space, to a stochastic differential equation. Euler approximation at sampled points leads to a likelihood function, used in the Bayesian estimation of the thermal conductivity under different prior densities. An approach for generating latent observations over time in points where the temperature is not acquired is also included. Finally, the methodology is experimentally validated, considering a heated piece of polymethyl methacrylate (PMMA) with temperature measurements available in few points of the material and acquired at high frequency.

In the second approach a Bayesian setting is developed to infer unknown parameters that appear into initial-boundary value problems for parabolic partial differential equations. The realistic assumption that the boundary data are noisy is introduced, for a given prescribed initial condition. We show how to derive the global likelihood function for the forward problem, given some measurements of the solution field subject to Gaussian noise. Given Gaussian priors for the time-dependent Dirichlet boundary values, we marginalize out analytically the global likelihood using the linearity of the discretized solution. This approach is fully implemented in the case of the heat equation where the thermal diffusivity is the unknown parameter. We assume that the thermal diffusivity parameter can be modeled a priori through a lognormal random variable or by means of a space-dependent stationary lognormal random field. Synthetic data are used to carry out the inference. We exploit the concentration of the posterior distribution of the thermal diffusivity, using the Laplace approximation and therefore avoiding costly MCMC computations. Expected information gains and predictive posterior densities for observable quantities are numerically estimated for different experimental setups.