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Abstract

Title: Fréchet Means and Procrustes Analysis in Wasserstein Space

Given a collection of diffuse random measures in R^d, we consider the problems of: constructing their optimal multicoupling in a mean square sense (a.k.a. Procrustes analysis); and of determining their Fréchet mean with respect to the Wasserstein metric. Though these problems admit explicit solutions when d=1, they are considerably more challenging when d>1. To attack them, we exploit the geometry of Wasserstein space, and specifically the relation between its tangent bundle and the class of optimal coupling maps. This allows us to determine the gradient of the Fréchet functional and show that the two problems at hand represent two sides of the same coin: gradient descent for the Fréchet mean reduces to a Procrustes algorithm, requiring only successive solution of pairwise coupling problems, thus being practically feasible; and the optimal maps obtained as a bye-product can be used to explicitly construct the optimal multi-coupling. We show how these results can be used in order to consistently solve the problem of registration of warped spatio-temporal point processes in an entirely nonparametric fashion, by means of appropriate regularisation. (Based on joint work with Yoav Zemel, EPFL).