Testing Uncovered Interest Parity under the Assumption of Liquidity Premia\textsuperscript{1}

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Abstract
The present study investigates whether liquidity premia can explain deviations from uncovered interest parity. For that purpose I modify a representative agent asset-pricing model by assuming that investors value liquidity services which are unique features of U.S. Treasuries. Further the assumption that domestic and foreign bonds are perfect substitutes is relaxed. Estimation results for U.S. and U.K. data provide support for the hypothesis that investors’ valuation for U.S. Treasuries’ liquidity contributes to explain deviations from uncovered interest parity. In contrast to most forward premium regression estimations, I find a positive association between the expected depreciation rate of the U.S. currency relative to the UK currency and the U.S.-U.K. Treasury yield spread.

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\textit{Keywords:} uncovered interest parity, exchange rates, key currency, liquidity premium, asset pricing.

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1 Introduction

Uncovered interest parity (UIP) implies that, under the assumption that covered interest parity holds, the differential between two countries’ risk-free interest rates is an estimate of future changes in the spot exchange rate. If expectations are rational then the interest differential should be an unbiased predictor of future bilateral exchange rate changes. There is a large branch of empirical works in the literature on testing UIP which employs forward premium regressions. These studies regress realized exchange rate changes on the interest differential or forward premium resp. between two countries. Under rational expectations and risk neutrality, UIP predicts this regression to yield a positive coefficient of unity on the forward premium. The empirical failure of UIP has been documented by various evidence from forward premium regressions. The widely quoted result by Froot (1990) finds that the average estimate of the mentioned coefficient across 75 published studies is -0.88. Furthermore, only a few estimates are positive but none is equal or greater than unity. This result is known as the forward premium puzzle. It implies that the forward premium predicts future changes in the spot exchange rate which are inconsistent with UIP, in terms magnitude and in terms of the direction of the movement.

The present study investigates whether liquidity premia can explain deviations from UIP. Specifically, in this study I examine the impact of liquidity premia on international interest rate differentials, namely the U.S.-U.K. Treasury yield spread.

UIP is a key no-arbitrage condition in international bond markets. Canzoneri et al. (2013) explain deviations from UIP by relaxing the assumption that risk-free bonds which are denominated in different currencies are perfect substitutes. Specifically, home and foreign bonds are imperfect substitutes for money in each countries’ transaction technology. Canzoneri et al. (2013) argue that the U.S. dollar’s role as a key currency in the international monetary system is the reason for the relatively low U.S. Treasuries’ interest rates. It is pointed out that U.S. Treasuries facilitate transactions in a number of ways: they serve as collateral in many financial markets, banks hold them to manage the liquidity of their portfolios, individuals hold them in money market accounts that offer checking services, and importers and exporters hold them as transaction balances. Therefore, the liquidity of U.S. Treasury bonds is interpreted as a non-pecuniary return to investors which poses the rationale for why U.S. Treasuries will be held at a discount. Hence, the key currency

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4As pointed out by Frankel (1982), employing regression analysis will test the joint hypothesis of the UIP’s implications regarding expected exchange rate changes together with unbiased expectations both to hold. This is denoted as the "unbiasedness hypothesis".
5For seminal survey articles about the empirical work on testing UIP see Hodrick (1987), Froot (1990), and Engel (1996). See Burnside et al. (2006) and Chinn (2006) for recent empirical studies and Engel (2013) for a recent survey.
feature of the U.S. dollar can contribute to the explanation of deviations from UIP.

A recent study by Krishnamurthy and Vissing-Jorgensen (2012) provides evidence that the corporate-Treasury bond yield spread is to a significant extent driven by the total amount of U.S. Treasuries outstanding which is proxied by the government Debt-to-GDP ratio (i.e. the market value of publicly held U.S. government debt to U.S. GDP). They argue that investors value certain features of U.S. Treasuries, namely their liquidity and their "absolute security of nominal return". This affects prices of Treasuries and drives down their yields compared to assets that do not to the same extent share these features. As a theoretical rationale for this observation Krishnamurthy and Vissing-Jorgensen (2012) assume that the holder of a U.S. Treasury security obtains some services and gains to the subjective level of well-being. Those benefits are summarized as "convenience yield" which directly contribute to investors' utility and lead U.S. Treasuries to have a lower yield than they would have in a standard asset-pricing framework.

I follow Krishnamurthy and Vissing-Jorgensen (2012) by modifying a standard representative agent asset-pricing model by allowing agents to derive utility directly from holdings of U.S. Treasuries. In a next step I derive no-arbitrage conditions for the international bond market which are implied by the modified model. Here I follow Canzoneri et al. (2013) by relaxing the assumption that U.S. domestic bonds and foreign bonds are perfect substitutes. Specifically, it is assumed that U.S. Treasuries provide unique liquidity services. Therefore, the model-implied no-arbitrage conditions allow for liquidity premia induced by the U.S. dollar’s postulated key currency feature. The no-arbitrage conditions are further derived for the cases of explicitly accounting for foreign exchange risk and price risk, and for neglecting these risk premia. I employ regression analysis to empirically test whether the model-implied no-arbitrage conditions for U.S. data and U.K. data can explain deviations from UIP. In this context I follow Fuhrer (2000) by assuming that the households’ expectations regarding the dynamics of consumption and the depreciation rate of the domestic currency can be described by an unconstrained vector autoregression.

I find that investors’ valuation for U.S. Treasuries’ liquidity contributes to explain deviations from UIP. Further, estimation results imply a positive association between the expected depreciation rate of the U.S. currency relative to the U.K. currency and the U.S.-U.K. Treasury yield spread or forward premium. However, the point estimate of the coefficient still is below unity.

There have been many attempts to account for departures from UIP. One of the most influential of these is Fama (1984) who attributes deviations of realized exchange rate changes from UIP to a time-varying risk premium. However, studies like Backus, Foresi

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6 Specifically, a risk premium arises in such models due to the degree to which the exchange rate return covaries with consumption growth.
and Telmer (2001) and Brandt, Cochrane and Santa-Clara (2006) confirm the result that risk premia cannot resolve the forward premium puzzle while assuming standard preferences. There are recent studies in this field which are able to account for deviations from UIP by assuming utility maximizing representative agents in home and foreign countries with non-standard preferences. E.g. Verdelhan (2010) employs a utility specification of external habit preferences, and Bansal and Shaliastovich (2010) use Epstein-Zin preferences. However, these studies rely on simulations of models which are calibrated to match a set of macroeconomic and financial data features. Further, small open economy models commonly include a UIP condition (see Gali and Monacelli (2005)). Authors like McCallum and Nelson (2000) and Kollmann (2005) add an exogenous UIP shock which is calibrated to align the model-implied volatility of exchange rates with the observed interest rate differentials. Justiniano and Preston (2010) conduct a Bayesian estimation of a small open economy model with an exogenous UIP shock. They find that the volatility in the real exchange rate is almost completely explained by a risk premium shock. This is interpreted as an extreme version of exchange rate disconnect. The generally bad empirical performance is attributed to the failure of the estimated model to link movements in the exchange rate with macroeconomic fundamentals. Therefore, addressing the issue of exchange rate disconnect is regarded as a key to improve the model’s quantitative performance.

This paper is organized as follows. In section 2 I derive the modified asset pricing model, no-arbitrage conditions for the international bond market, and specify regression models which are estimated to test the no-arbitrage condition’s implications. Section 3 presents estimation results. Section 4 concludes.

2 Yield spread model

In the following section I modify an asset pricing model under the assumption that U.S. Treasuries’ liquidity services are valued by investors. This is done along the lines of Krishnamurthy and Vissing-Jorgensen (2012). Spreads between the yields of U.S. Treasuries and foreign Treasury securities which do not provide liquidity services are then explained by a no-arbitrage condition. The goal is to obtain a model of spread determinants which can be empirically tested for its ability to explain observed international Treasury yield spreads.

2.1 Household’s problem

A representative household is assumed to maximize the expected sum of a discounted stream of utilities
\begin{equation}
E_0 \sum_{t=0}^{\infty} \beta^t u \left( c_t, \nu \left( b_t, GDP_t, \xi_t \right) \right),
\end{equation}

subject to the budget constraint

\begin{equation}
P_t c_t + P^T_t b_t + X_t P^T_t b^*_t \leq P_t y_t + P^T_{t-1} b_{t-1} + X_t P^T_{t-1} b^*_t,
\end{equation}

where $E_0$ is the expectation operator conditional on the information set in the initial period, and $\beta \in (0,1)$ is the subjective discount factor. The utility function is specified by $u_t = \frac{1}{1-\sigma} c_t^{1-\sigma} + \nu (b_t, GDP_t, \xi_t)$, with $\sigma \geq 1$, where $c_t$ denotes consumption and $\nu (\cdot)$ represents the agent’s gained convenience yield. I follow Krishnamurthy and Vissing-Jorgensen (2012) by assuming that convenience yields are a function of the U.S. gross domestic product (GDP) and the investor’s holdings of real U.S. Treasuries $b_t$.\textsuperscript{7} The latter further captures the assumption by Canzoneri et al. (2013), that the U.S. dollar’s role as a key currency in the international monetary system induces holdings of U.S. Treasuries to yield unique non-pecuniary returns to the investors. The term $\xi_t$ is a preference shock. Following Krishnamurthy and Vissing-Jorgensen (2012) the convenience yield function $\nu (\cdot)$ is concave with $\nu' (\cdot) > 0$, and $\nu'' (\cdot) < 0$. Further, $\nu (\cdot)$ shall be homogenous of degree one in $GDP_t$ and $b_t$.\textsuperscript{8} The household earns a real endowment income $y_t$, and carries wealth into the next period by investing into nominal holdings of U.S. Treasuries $P^T_t b_t$, and by investing into nominal holdings of the foreign country’s Treasuries $P^T_t b^*_t$. In order to measure the purchasing power of a foreign currency pay-off in a particular period $t$, the nominal exchange rate $X_t$ is introduced. The exchange rate is measured as the price of foreign currency in units of domestic currency at time $t$. Assume for simplicity that the agent buys zero coupon discount bonds which pay out one unit of currency when being held to maturity. The aggregate price level at date $t$ is denoted by $P_t$. The nominal prices for one-period investments into U.S. Treasuries, and into the foreign country’s Treasuries are denoted as $P^T_t$, and $P^T_t$. Real holdings of foreign Treasuries are denoted as $b^*_t$.

Maximizing the objective function (1) subject to the budget constraint (2) leads for given initial values and non-negativity constraints for $b_t$, and $b^*_t$ to the following first order conditions for consumption $c_t$, and investments into U.S. Treasuries $b_t$, and foreign Treasuries $b^*_t$:

\begin{equation}
c_t^{-\sigma} = \lambda_t,
\end{equation}

\textsuperscript{7}Specifically, Krishnamurthy and Vissing-Jorgensen (2012) assume that convenience yields are driven by a set of macroeconomic factors which will influence the household’s level of well-being. The U.S. GDP acts as a shortcut to capture these factors.

\textsuperscript{8}Hence $\nu (\cdot)$ can be transformed in the following manner: $\nu (b_t, GDP_t, \xi_t) \equiv \nu \left( \frac{b_t}{GDP_t}, \xi_t \right) GDP_t$. 

\[
\nu'(\cdot) \frac{P^T_{t+1}}{P_t} + \beta E_t \left[ \lambda_{t+1} \frac{P^T_{t+1}}{P_{t+1}} \right] = \lambda_t \frac{P^T_t}{P_t}, \tag{4}
\]

\[
\beta E_t \left[ \lambda_{t+1} \frac{X_{t+1}P^T_{t+1}}{P_{t+1}} \right] = \lambda_t \frac{X_t P^T_{t+1}}{P_t}, \tag{5}
\]

and (2) holding with equality, and the transversality conditions

\[
\lim_{j \to 0} \beta^j E_t \left( \lambda_{t+j} P^T_{t+j} \right) = 0, \quad \text{and} \quad \lim_{j \to 0} \beta^j E_t \left( \lambda_{t+j} X_{t+j} P^T_{t+j} \right) = 0.
\]

The stochastic discount factor for nominal payoffs is denoted as

\[
M_{t+1} = \frac{P^T_{t+1}}{P_t},
\]

such that (4), and (5) combined with (3) imply

\[
P^T_t = \frac{E_t \left[ M_{t+1} P^T_{t+1} \right]}{1 - \nu'(\cdot) / c_t^{-\sigma}}, \tag{6}
\]

\[
P^T_{t+1} = \frac{E_t \left[ M_{t+1} (1 + q_{t+1}) P^T_{t+1} \right]}{1 - \nu'(\cdot) / c_t^{-\sigma}}, \tag{7}
\]

where I denote \((1 + q_{t+1}) = X_{t+1} / X_t\), as the gross return on holding one unit of foreign currency. Equation (6) requires that under the assumption that U.S. Treasuries provide liquidity services as an argument of the investor’s utility function, increasing the amount of U.S. Treasuries held, will decrease their price \(P^T_t\). Specifically, increasing the stocks of U.S. Treasuries will lower the investor’s willingness to pay for another unit of such assets. This is due to the assumption of \(\nu'(\cdot)\) being a concave function of \(b_t\). Note that foreign Treasuries do not provide liquidity services.

### 2.2 No-arbitrage condition without risk premium

In this section I derive the no-arbitrage condition for the international bond market under the assumption that financial markets are complete.\(^9\) Note that I consider zero coupon discount bonds. Further, I assume that there is no price risk or default risk for the two Treasury bonds under consideration. Therefore, \(P^T_{t+1} = P^T_{t+1} = 1\). Hence, \(P^T_t = \frac{1}{R^T_t}\), and

\[
P^T_{t+1} = \frac{1}{R^T_{t+1}},
\]

where \(R^T_t\) and \(R^T_{t+1}\) are the risk-free gross returns. Equations (4) and (5) can now be written as

\[
\nu'(\cdot) \frac{1}{\lambda_t R^T_t} + \beta E_t \left[ \lambda_{t+1} \frac{1}{\pi_{t+1}} \right] = \frac{1}{R^T_t}, \tag{8}
\]

\[
\beta E_t \left[ \lambda_{t+1} \frac{X_{t+1}}{\lambda_t X_t} \frac{1}{\pi_{t+1}} \right] = \frac{1}{R^T_{t+1}}, \tag{9}
\]

\(^9\)In a similar way Gali and Monacelli (2005) derive the UIP condition for a small open economy model.
where inflation is given by $\pi_{t+1} = P_{t+1}/P_t$. Now denote the left-hand side of (9) as stochastic discount factor in terms of purchasing power in the foreign currency:

$$M_{t+1}^* = \beta \lambda_{t+1} \frac{1}{\pi_{t+1}} \frac{X_{t+1}}{X_t} = M_{t+1} \frac{X_{t+1}}{X_t}.$$ 

Substituting out in (8) yields

$$\nu'(\cdot) \frac{1}{c_t} + E_t \left[ M_{t+1}^* \frac{X_t}{X_{t+1}} \right] = \frac{1}{R_t^T}.$$ 

Given that $1/R_t^{T*} = E_t [M_{t+1}^*]$, is the foreign currency rate of return on a nominally risk-free Treasury, the no-arbitrage condition can be derived

$$\frac{R_t^T}{R_t^{T*}} E_t \left[ \frac{X_t}{X_{t+1}} \right] = 1 - \frac{\nu'(\cdot)}{c_t^{-\sigma}}.$$ 

Taking the logarithm of the former expression yields then

$$r_t^T - r_t^{T*} = E_t [q_{t+1}] - \nu'(\cdot) / c_t^{-\sigma}. \quad (10)$$

This approximation uses that $\ln(1 - y) \approx y$, for small $y$. Note that $E_t [x_{t+1}] - x_t = E_t [q_{t+1}]. \quad 10$ Equation (10) implies that the investors’ marginal valuation for the U.S. Treasuries’ liquidity induces a deviation from UIP. Specifically, this equation implies a positive relation between the holdings of U.S. Treasuries and the interest rate differential $r_t^T - r_t^{T*}$. Increasing the holdings of U.S. Treasuries decreases the investor’s marginal valuation for any further unit of U.S. Treasuries $\nu'(\cdot)$. This in turn reduces the U.S. Treasuries’ prices and will therefore drive up the expected returns.

### 2.3 Yield spread model with risk premium

In a next step this section employs the modified asset pricing model to explain international Treasury bond yield spreads while accounting for price risk and foreign exchange risk. I follow Krishnamurthy and Vissing-Jorgensen (2012)\textsuperscript{11} by computing the $\tau$-period yields for U.S. Treasury debt securities $i_{t,T}^T$, and for foreign Treasury debt securities $i_{t,T}^{T*}$:

$$i_{t,T}^T = -\frac{1}{\tau} ln P_{t}^{T}, \text{ and } i_{t,T}^{T*} = -\frac{1}{\tau} ln P_{t}^{T*},$$

\textsuperscript{10}Note that $x_t$ and $x_{t+1}$ are the logarithms of the period $t$ and period $t+1$ exchange rates, and that the net returns $r_t^T$ and $r_t^{T*}$ are the logarithms of the gross returns $R_t^T$ and $R_t^{T*}$.

\textsuperscript{11}This is applied by Backus, Foresi and Telmer (2001) to calculate prices of bonds with different maturities in the context of affine models of the term structure of interest rates.
where \( \tau \) is the number of periods to maturity. By this, the price of a zero coupon bond is converted into a continuously compounded zero coupon bond yield. Therefore, for discount bonds with \( P^T_\tau = P^{T*}_\tau = 1 \), the yield spread for securities with any number of periods to maturity \( \tau \), can be expressed as:

\[
i^{T}_{t, \tau} - i^{T*}_{t, \tau} = \frac{1}{\tau} \left( \ln P^{T*}_t - \ln P^T_t \right).
\]

Now plug in (6) for \( P^T_t \) and (7) for \( P^{T*}_t \):

\[
= \frac{1}{\tau} \left( \ln \left( E_t [M_{t+\tau} (1 + q_{t+\tau})] \right) - \ln \left( \frac{E_t [M_{t+\tau}]}{1 - \nu' (\cdot) / c_t^{-\sigma}} \right) \right),
\]

\[
\approx \frac{1}{\tau} \left( E_t [M_{t+\tau} (1 + q_{t+\tau})] - E_t [M_{t+\tau}] - \nu' (\cdot) / c_t^{-\sigma} \right).
\]

This approximation uses that \( \ln (1 + y) \approx y \), for small \( y \). Denote the yield spread as \( \Delta i_{t, \tau} = i^{T}_{t, \tau} - i^{T*}_{t, \tau} \), and rearrange

\[
\Delta i_{t, \tau} = \frac{1}{\tau} E_t [M_{t+\tau}] E_t [q_{t+\tau}] + \frac{1}{\tau} \text{cov}_t (M_{t+\tau}, q_{t+\tau}) - \frac{1}{\tau} \nu' (\cdot) / c_t^{-\sigma}. \tag{11}
\]

Equation (11) implies that the \( \tau \)-period spread between the yield of a U.S. Treasury security and the yield of a foreign Treasury security with remaining term to maturity \( \tau \), is determined by the product of the expected \( t + \tau \)-periods-ahead stochastic discount factor times the expected \( t + \tau \)-periods ahead exchange rate growth rate, the covariance between the \( t + \tau \)-periods-ahead stochastic discount factor and the \( t + \tau \)-periods-ahead exchange rate growth rate, and the period \( t \) marginal convenience yield of U.S. Treasuries divided by \( c_t^{-\sigma} \). The former two terms reflect the foreign exchange risk premium which arises due to the comovement of the future expected spot exchange rate growth rate with the expected change in the stochastic discount factor. Note that the third term on the right-hand side of (11) reflects the modification of the standard asset pricing model by the assumption that investors value features of U.S. Treasuries which are unique to them. By the assumption of the U.S. dollar to be the key currency, these features are not shared with any other Treasury debt security issued by any other country.

### 2.4 Estimation strategy

The purpose of the present study is to test the hypothesis that investors value the unique liquidity of U.S. Treasuries which leads to deviations from UIP. This is done by investigating whether the marginal convenience yield terms in (10) and (11) significantly contribute to the explanation of the observed yield spreads for U.S. Treasuries compared to U.K.
Treasury debt securities.\(^{12}\) For that purpose I specify the following regression models:

\[
\Delta t^{U.S.UK} = \alpha_{np} + \beta_{1np}^2 \log\left(\frac{Debt_t}{GDP_t}\right) + \beta_{2np}^2 q_t + \varepsilon_t np,
\]

\[
\Delta t^{U.S.UK} = \alpha_{rp} + \beta_{1rp}^2 \log\left(\frac{Debt_t}{GDP_t}\right) + \beta_{2rp}^2 \left(\frac{\hat{M}_{t+1}}{\text{cov}(M, q)}\right) + \varepsilon_t^p,
\]

where \(\Delta t^{U.S.UK}\) denotes the spread between the yields of a U.S. Treasury and a U.K. Treasury with same maturity length, and \(\varepsilon_t np\) and \(\varepsilon_t^p\) denote error terms. The dependent variable is a quarterly yield spread measured in percentage points. Specifically, I use quarterly data and 3-month Treasury bill yields.\(^{13}\) The superscript \(np\) denotes the estimation model for the no-arbitrage condition \((10)\), and the superscript \(rp\) denotes the estimation model for the yield spread model \((11)\) which accounts for risk premia. The proxy for the marginal convenience yield which is divided by marginal utility of consumption in the equations \((10)\) and \((11)\) is the logarithm of the face value of the outstanding stock of U.S. Treasuries, scaled by U.S. GDP. This proxy is denoted as \(\log\left(\frac{Debt_t}{GDP_t}\right)\). A log functional form is used because it provides a good fit and requires estimation of only one parameter. Further, the interpretation of a regression coefficient for a log independent variable on a dependent variables denoted in percentage points is more convenient.

I follow Fuhrer (2000) by assuming that the dynamics of the stochastic discount factor and the growth rate of the exchange rate can be described by an unconstrained vector autoregression.\(^{14}\) In particular, the vector autoregression is used to generate the households' forecasts of the future changes in consumption and inflation, which are required to calculate the expected changes in the stochastic discount factor \(\hat{M}_{t+1}\), and forecasts of the exchange rate \(\hat{q}_{t+1}\). These variables enter the right-hand sides of the yield spread regression models \((12)\) and \((13)\).\(^{15}\) Note that I regard the covariance between the stochastic discount factor and the growth rate of the exchange rate \(\text{cov}(M, q)\) which enters the right

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\(^{12}\)For the present study I focus on U.S. and U.K. data because Treasury debt securities issued in both countries can be regarded as close substitutes apart from the postulated key currency feature of the U.S. dollar. Specifically, financial market integration between both countries is intense and trading volumes of U.S. Treasuries and U.K. Treasuries are large.

\(^{13}\)See Appendix A for a description of the data.

\(^{14}\)This approach is similar to the one applied in Campbell and Shiller (1987) for present value models, such as consumption functions that relate consumption, income, and interest rates. Campbell and Shiller (1987) point out that employing an unconstrained vector autoregression to generate forecasts implies the choice of the information set which includes all relevant information of market participants at the time when expectations are formed.

\(^{15}\)Canzoneri et al. (2007) use this approach to generate the forecasts of the future changes in consumption and inflation. Their aim is to compute implied consumption Euler equation rates under a number of different preference specifications and compare them to observed nominal and real market rates.
hand side of (13), as being constant.\textsuperscript{16}

The VAR is estimated from 1985:Q3 to 2008:Q1 on quarterly U.S. data. Following Fuhrer (2000) the VAR is estimated for log per capita real nondurable goods and services consumption, the log per capita real disposable income, the effective federal funds rate, the log per capita real nonconsumption GDP, the log change in the price index for nondurables and service consumption, and a commodity price index. Further, I follow Eichenbaum and Evans (1995) by additionally considering the U.S. dollar relative to U.K. pound exchange rate for the VAR model estimation.\textsuperscript{17}

Note that by employing this VAR model to generate households’ forecasts, the exogeneity assumption for the variables $\tilde{q}_{t+1}$ and $\tilde{M}_{t+1}$ in the regression models (12) and (13) might be violated. In this case OLS estimates would be invalid. To justify the use of OLS for the purpose of the present study it is assumed that the forecasts of $\tilde{q}_{t+1}$ and $\tilde{M}_{t+1}$ are contemporaneously uncorrelated with the disturbances $\varepsilon_{t}^{np}$ and $\varepsilon_{t}^{rp}$.\textsuperscript{18}

By estimating (12) and (13) the intention is to test the following hypotheses:

\textbf{Hypothesis 1} The yield spread models (10) and (11) require that an increase in the U.S. Debt to GDP ratio which is a proxy for the holdings of liquid U.S. Treasuries, increases the observed U.S.-U.K. Treasury yield spreads. Hence, the regression analysis would provide support in favor of the assumption that investors value unique liquidity features of U.S.

\textsuperscript{16} The conditional moments are obtained from a VAR($p$) model with $k$ endogenous variables which are elements of the vector $\tilde{Y}_{t} = (Y_{1,t}, \ldots, Y_{k,t})$:  
\[
\tilde{Y}_{t} = A_{0} + A_{1} \tilde{Y}_{t-1} + \ldots + A_{p} \tilde{Y}_{t-p} + \tilde{e}_{t}, \\
\tilde{e}_{t} \sim IID (0, \Sigma_{e}),
\]

where $A_{0}$ and $\tilde{e}_{t}$ are $k \times 1$ vectors of the constant terms and the independent and identically distributed random error terms. The $k \times k$ matrices $A_{1}$ and $A_{p}$ contain the regressors’ parameters. The conditional expectations for the $h$-periods-ahead consumption and exchange rate are derived by computing  
\[
E_{t} \tilde{Y}_{t+h} = \hat{A}_{0} + \hat{A}_{1} \tilde{Y}_{t+h-1} + \ldots + \hat{A}_{p} \tilde{Y}_{t+h-p},
\]

where $\hat{A}_{0}$, is the vector of the regression intercepts, and $\hat{A}_{1}$, and $\hat{A}_{p}$ contain the estimated regression coefficients. The conditional second moments are given by the elements of the estimated covariance matrix  
\[
\hat{\Sigma} = \frac{T}{T - kp - 1} \hat{U} \hat{U}',
\]

where $\hat{U}$ is the $k \times T$ matrix of the regression residuals.

\textsuperscript{17} See Appendix A for a detailed description of the data.

\textsuperscript{18} Following Mehra and Minton (2007), OLS has been employed in contributions like Orphanides (2001) and Boivin (2006), to estimate forward-looking Taylor rules using the Federal Reserve Board of Governors’ Greenbook forecasts. As further pointed out, the use of OLS requires the assumption that the Greenbook forecasts are uncorrelated with the regression error which is interpreted as a monetary policy rate shock. Boivin (2006) argues that this exogeneity assumption can not be directly verified but is implicitly made by studies like Orphanides (2001) when using OLS to estimate forward-looking Taylor rules with forecast data. As the present study is conceptually similar I follow these authors and assume that the VAR forecasts are uncorrelated with the disturbances $\varepsilon_{t}^{np}$ and $\varepsilon_{t}^{rp}$. 

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Treasuries if point estimates for the regression coefficients would imply that $\beta_{1p}^{np} > 0$, and $\beta_{1}^{rp} > 0$.

This would imply that liquidity premia can contribute to explain deviations from UIP. Further, I investigate whether foreign exchange risk provides in this context a significant contribution to the explanation of the observed variation in the U.S.-U.K. Treasury yield spread.

**Hypothesis 2**  
*The yield spread model (13) provides a better empirical fit. Employing the regression model (12) to explain the spread between U.S. Treasury yields and U.K. Treasury yields neglects important information.*

### 3 Empirical results

Equations (12) and (13) are estimated on quarterly data ranging from 1985:Q3 to 2008:Q1. This data sample is chosen for the present analysis as it covers roughly the period on which recent empirical work testing the UIP condition is estimated.\(^{19}\) The dependent variable is the spread between the 3-month U.S. Treasury bill yield and the U.K. Treasury bill yield with the same maturity length.

Estimation results are summarized in Table I. The first column of Panel A presents coefficient estimates for the regression of the U.S.-U.K. Treasury yield spread on the measure for U.S. Treasury holdings $\log (\text{Debt}_t/GDP_t)$, the expected next quarter’s growth rate of the exchange rate $\hat{q}_{t+1}$, and a constant term. The mean value of the U.S.-U.K. Treasury bill yield spread is $-266$ basis points (bp) for the sample period 1985:Q3 to 2008:Q1. The coefficient of 11.18 on the $\log (\text{Debt}_t/GDP_t)$ variable implies that a one percentage point increase of the average U.S. Debt-to-GDP ratio, increases the U.S.-U.K. Treasury bill yield spread by 21 bp. Note that a one standard deviation increase in the U.S. Debt-to-GDP ratio, from its mean value of 0.52 to 0.65, increases the U.S.-U.K. Treasury bill yield spread by 249 bp. From the perspective of the no-arbitrage condition (10) one would argue that such an increase in the holdings of Treasuries which are denominated in the key currency, decreases the investors’ valuation and willingness to pay for an other unit of such Treasuries. This in turn drives up the yield of a U.S. Treasury bill compared to the yield of a U.K. Treasury bill. This finding is consistent with Hypothesis 1 and statistically significant. Further, it implies that U.S. Treasury supply is an important determinant of the spread. The covariate $\hat{q}_{t+1}$ is in this setting estimated to be significantly related to the spread. The point estimate for $\beta_2^{pp}$ is 0.34 which implies that an expected depreciation of the U.S. currency relative to the U.K. currency is positively related to an increase in the spread.

\(^{19}\)See Burnside et al. (2006) and Chinn (2006).
U.S.-U.K. Treasury yield spread. Under the standard specification of the forward premium regression model one would test the hypothesis whether the estimated coefficient on $\hat{q}_{t+1}$ is unity. This however, is not found in the present study, but in contrast to most empirical studies on forward premium regressions, the point estimate of the coefficient in the present study is significantly larger zero. In the second column of Panel A the estimated coefficients are presented for a regression where the measure for U.S. Treasury holdings is not included. Results imply that $\hat{q}_{t+1}$ has in this case no significant impact on the spread. Therefore, the positive association of $\hat{q}_{t+1}$ with the spread found for the regression presented in the first column of Panel A, depends on the inclusion of $\log \left( \frac{\text{Debt}_t}{\text{GDP}_t} \right)$ as covariate to the estimation model. Further, including the $\log \left( \frac{\text{Debt}_t}{\text{GDP}_t} \right)$ regressor sharply increases the $R^2$ measure.

In the first column of Panel B results are shown for estimating the regression model (13). In this case the U.S.-U.K. Treasury yield spread is regressed on the proxy for the expected foreign exchange risk $\hat{M}_{t+1} \hat{q}_{t+1} + \text{cov} \left( M, q \right)$, instead of the expected next quarter’s growth rate of the exchange rate $\hat{q}_{t+1}$. Further, a constant and the measure for U.S. Treasury holdings $\log \left( \frac{\text{Debt}_t}{\text{GDP}_t} \right)$ are included. Again, estimating the model with the $\log \left( \frac{\text{Debt}_t}{\text{GDP}_t} \right)$ regressor increases the $R^2$ measure. Further, by comparison with the results presented in the second column of Panel B, it appears that by inclusion of the U.S. Treasury holdings proxy the coefficient on the proxy for the foreign exchange risk becomes significant. However, the size of the estimated coefficient implies a small effect of foreign exchange risk on the spread. Further, comparing the results across the first columns of Panel A and Panel B shows that the values of the point estimates for the coefficients on $\log \left( \frac{\text{Debt}_t}{\text{GDP}_t} \right)$, the regression constants, and the values of the $R^2$ measures lie very close together. Hence, the proxy for foreign exchange risk does not seem to contain important information for the U.S.-U.K. Treasury yield spread regression. Hence, I consider Hypothesis 2 to be rejected by this result.

4 Conclusion

For the present paper I modified a representative agent asset-pricing model by assuming that investors value liquidity services which are unique features of U.S. Treasuries. Further, the assumption that U.S. domestic bonds and foreign bonds are perfect substitutes was relaxed. In a next step model-implied no-arbitrage conditions for the international bond market were derived. These are interpreted as UIP conditions which are adjusted for liquidity premia. Estimation results provide support for the hypothesis that investors value the liquidity of U.S. Treasuries which yields a significant contribution to the explanation of the U.S.-U.K. 3-month Treasury bill yield spread. This implies that investors’
valuation for U.S. Treasuries’ liquidity contributes to explain deviations from UIP. Estimation results however, imply that foreign exchange risk can only explain a very low share of the observed variation in the U.S.-U.K. 3-month Treasury bill yield spread. In contrast to most forward premium regression estimations I find a positive association between the expected depreciation rate of the U.S. currency relative to the U.K. currency and the U.S.-U.K. Treasury yield spread. However, the point estimate of the coefficient is below unity.
References


A Data

U.S.-U.K. Treasury yield spread: This variable is constructed as the percentage spread between the U.S. Treasury bill yield for 3-month Treasuries extracted from the Federal Reserve of St. Louis’ FRED database (series TB3MS), and the U.K. Treasury bill yield with the same maturity length from Datastream (series UKTBTND).

Debt/GDP: This variable is intended to proxy for the holdings of U.S. Treasuries scaled by U.S. GDP. Here I use time series Data on the total amount of Treasury securities outstanding from Datastream (series USSECMNSA). U.S. GDP data is extracted from the FRED database (series GDP).

VAR model: The vector of the VAR model’s endogenous variables is given by
\[ \tilde{Y}_t = \left( c_t, \pi_t, y_t^{Dis}, i_t^{FED}, p_t^{Ind}, (y_t - c_t), X_t \right). \]

The endogenous variables are calculated using FRED data:

per capita real nondurable goods and services consumption:
\[ c_t = \frac{PCNDGC96_t + PCESVC96_t}{POP_t}, \]

inflation, measured by the log change in the price index for nondurables and service consumption:
\[ \pi_t = \log \left( \frac{PCND_t + PCESV_t}{PCNDGC96_t + PCESVC96_t} \right) \]
\[ - \log \left( \frac{PCND_{t-1} + PCESV_{t-1}}{PCNDGC96_{t-1} + PCESVC96_{t-1}} \right), \]

per capita real disposable income:
\[ y_t^{Dis} = \frac{DPIC96_t}{POP_t}, \]

the effective federal funds rate, \( i_t^{FED} = FEDFUNDS_t \),

a commodity price index, \( p_t^{Ind} = PPIIDC_t \),

the nominal U.S. Dollar to British Pound exchange rate \( X_t = EXUSUK_t \),
per capita real nonconsumption GDP:

$$(y_t - c_t) = \frac{GDP_{96_t}}{POP_t} - \frac{PCECC_{96_t}}{POP_t}.$$
Table 1: Impact of US Debt/GDP on U.S.-U.K. Treasury bills yield spread

<table>
<thead>
<tr>
<th>Period</th>
<th>1985:Q3 - 2008:Q1</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A)</td>
<td>(B)</td>
<td></td>
</tr>
<tr>
<td>$\log(\text{Debt/GDP})$</td>
<td>11.180</td>
<td>11.122</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[5.491]</td>
<td>[5.354]</td>
<td></td>
</tr>
<tr>
<td>$\hat{\eta}_{t+1}$</td>
<td>0.339</td>
<td>0.091</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[2.526]</td>
<td>[0.520]</td>
<td></td>
</tr>
<tr>
<td>$\hat{M}<em>{t+1}\hat{\eta}</em>{t+1}$</td>
<td>0.002</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[2.200]</td>
<td>[0.223]</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.263</td>
<td>0.004</td>
<td>0.250</td>
</tr>
<tr>
<td>N</td>
<td>89</td>
<td>89</td>
<td>89</td>
</tr>
</tbody>
</table>

Notes: The sample period is 1985:Q3 - 2008:Q1. t-statistics are reported in brackets.