Invariant coordinate selection (ICS) and multivariate location, scatter, skewness and kurtosis

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The classical measures of univariate location and scale are the mean and variance (or standard deviation), and the classical skewness and kurtosis measures are given by standardized third and fourth moments, respectively. We extend the classical measures to the multivariate case, and show how they can be used to construct an \textit{invariant coordinate system} (ICS) \citep{Tyler2009}. Asymptotical results for multivariate skewness and kurtosis measures (and for the transformation matrix) are outlined.

It is also shown that any two multivariate location measures (vectors) and any two multivariate scatter measures (matrices) may be used in a similar way to transform the observations to an invariant coordinate system. If the two scatter matrices have the so called independent property, a solution to the \textit{independent component analysis} (ICA) problem is obtained.

There are several possible uses of the ICS: Constructing of multivariate invariant/equivariant nonparametric tests and estimates based on the transformation and retransformation technique, hunting for clusters and outliers, reduction of dimension, etc. The multivariate measures of skewness and kurtosis can be also used to distinguish between a wide range of models that extend the multivariate normal model \citep{Nordhausen2009}.

The theory is illustrated with several examples. R-packages ICS and IC-SNP are available for practical data analysis. See \citep{Nordhausen2008a} and \citep{Nordhausen2008b}.

References


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